Removing Left-Recursions (1b)

```
1  ALGORITHM: RemoveLR
2    INPUT: CFG G = (V, Σ, R, S)
3    ASSUME: G has no ϵ-productions
4    OUTPUT: G' s.t. G' ≡ G, G' has no
5            indirect & direct left-recursions
6  PROCEDURE:
7    impose an order on V: ⟨(A_1, A_2, ..., A_n)⟩
8    for i: 1 .. n:
9      for j: 1 .. i − 1:
10       if ∃ A_i → A_j γ ∈ R ∧ A_j → δ_1 | δ_2 | ... | δ_m ∈ R then
11         replace A_i → A_j γ with A_i → δ_1 γ | δ_2 γ | ... | δ_m γ
12       end
13     for A_i → A_i α | β ∈ R:
14       replace it with: A_i → β A_i', A_i' → α A_i' | ϵ
```

Directly Left-Recursive CFG:

$$
\begin{array}{lcl}
Expr & \rightarrow & Expr + Term \\
 & | & Expr - Term \\
 & | & Term \\
Term & \rightarrow & Term * Factor \\
 & | & Term / Factor \\
 & | & Factor
\end{array}
$$

$$
\begin{array}{lcl}
Expr & \rightarrow & Term\ Expr' \\
Expr' & \rightarrow & + Term\ Expr' \\
 & | & - Term\ Expr' \\
 & | & \epsilon \\
Term & \rightarrow & Factor\ Term' \\
Term' & \rightarrow & * Factor\ Term' \\
 & | & / Factor\ Term' \\
 & | & \epsilon
\end{array}
$$

Removing Left-Recursions (2b)

Does the order of variables matter?

```
1  ALGORITHM: RemoveLR
2    INPUT: CFG G = (V, Σ, R, S)
3    ASSUME: G has no ϵ-productions
4    OUTPUT: G' s.t. G' ≡ G, G' has no
5            indirect & direct left-recursions
6  PROCEDURE:
7    impose an order on V: ⟨(A1, A2, ..., An)⟩
8    for i: 1 .. n:
9      for j: 1 .. i − 1:
10       if ∃ Ai → Ajγ ∈ R ∧ Aj → δ1 | δ2 | ... | δm ∈ R then
11         replace Ai → Ajγ with Ai → δ1γ | δ2γ | ... | δmγ
12       end
13     for Ai → Aiα | β ∈ R:
14       replace it with: Ai → βA'i, A'i → αA'i | ϵ
```

I	J
1 C	–
2 B	1 C
3 A	1, 2
	C B

(1) $[I, J = 2, 1]$

$B \to Cd$, $C \to At$

↳ $B \to Atd$

(2) $[I, J = 3, 2]$

$A \to Br$, $B \to Atd$

↳ $A \to Atdr$

Indirectly Left-Recursive CFG:

① $C \to At$

② $B \to Cd$ ($B \to Atd$)

③ $A \to Br$ ($A \to Atdr$)

Removing Left-Recursions (2c)

```
1  ALGORITHM: RemoveLR
2    INPUT: CFG G = (V, Σ, R, S)
3    ASSUME: G has no ϵ-productions
4    OUTPUT: G' s.t. G' ≡ G, G' has no
5            indirect & direct left-recursions
6  PROCEDURE:
7    impose an order on V: ⟨(A1, A2, ..., An)⟩
8    for i: 1 .. n:
9      for j: 1 .. i − 1:
10       if ∃ Ai → Ajγ ∈ R ∧ Aj → δ1 | δ2 | ... | δm ∈ R then
11         replace Ai → Ajγ with Ai → δ1γ | δ2γ | ... | δmγ
12       end
13     for Ai → Aiα | β ∈ R:
14       replace it with: Ai → βA'i, A'i → αA'i | ϵ
```

Indirectly Left-Recursive CFG:

① A	→	Ba	|	b	
② B	→	Cd	|	e	
③ C	→	Df	|	g	
④ D	→	f	|	Aa |	Cg

D → f | Aa | Cg

Replace A by Ba and b

D → Baa | ba | f | Cg

Replace B by Cd and e

D → Cdaa | eaa | baf | Cg

Replace C by Df and g

D → Dfdaa | gdaa | eaa | ba | Dfg | gg

Exercise: Remove two occurrences of direct LRs one by one.